Sum of first N odd numbers

There is an interesting property for the sum of first N odd numbers. For example sum of first 3 odd numbers, 1 + 3 + 5 = 9 which is 3^2. Take another example: sum of first 5 odd numbers, 1 + 3 + 5 + 7 + 9 = 25 which is 5^2.

This means that sum of first N odd numbers is N^2.

Any proof?. Of course there are algebraic proofs. But we are considering a geometric proof.

Algebraic Proof

Take the difference of squares of two adjacent counting numbers, N and N+1

$$(N+1)^2 - N^2 = N^2 + 2N + 1 - N^2 = 2N + 1$$

If this is said in words: The difference between square of an integer and next number is one more than double of that integer.

This means
$1^2 - 0^2$ = Difference between 0^2 and 1^2 will be 2 * 0 + 1 = 1
$2^2 - 1^2$ = Difference between 1^2 and 2^2 will be 2 * 1 + 1 = 3
$3^2 - 2^2$ = Difference between 2^2 and 3^2 will be 2 * 2 + 1 = 5
$4^2 - 3^2$ = Difference between 3^2 and 4^2 will be 2 * 3 + 1 = 7
$5^2 - 4^2$ = Difference between 4^2 and 5^2 will be 2 * 4 + 1 = 9

Adding all these:

$$1^2 - 0^2 + 2^2 - 1^2 + 3^2 - 2^2 + 4^2 - 3^2 + 5^2 - 4^2 = 1 + 3 + 5 + 7 + 9$$

On the left side the terms 1^2, 2^2, 3^2, 4^2 will be cancelled out and only 5^2 is left.

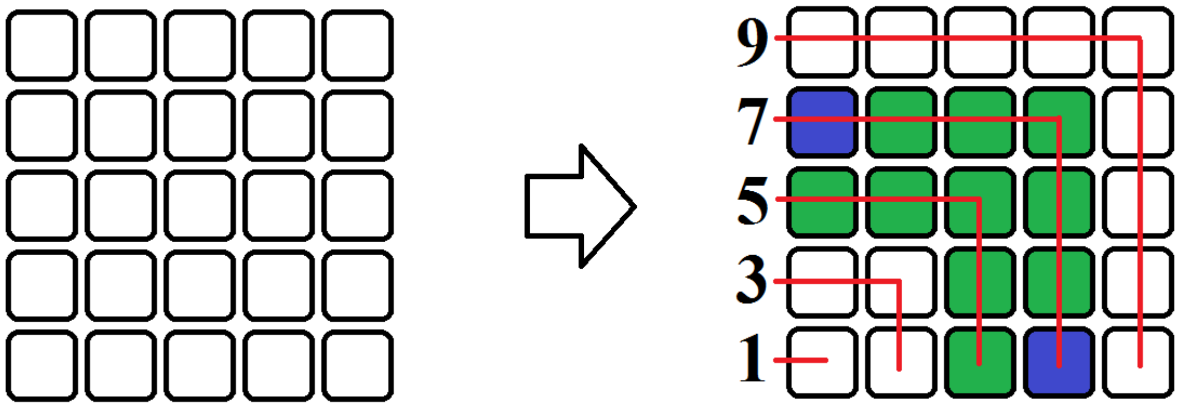
This means: $5^2 = 1 + 3 + 5 + 7 + 9$ proved.

It is proved for 5^2 but it can be easily extended to any number.

The above is a simple Algebraic Proof. But here I am giving a geometrical proof for this which give a different perspective for this theorem.

Geometric Proof

As done for Algebraic Proof we will do the Geometric proof for 5^2.



5 X 5 square

The above figure shows a 5 X 5 square. Obviously there are 25 small squares in the figure. These squares can be counted in a different way shown on the right side figure.

1. The small square on the bottom left side is counted, which is 1.
2. The squares covering the first one on top and right is counted, which total to 3.
3. The squares covering the second group on top and right is counted, which total to 5.
4. The squares covering the third group on top and right is counted, which total to 7.
5. The squares covering the fourth group on top and right is counted, which total to 9.

So total number of squares = 25 = 1 + 3 + 5 + 7 + 9 = sum of first 5 odd numbers.

We grouped the small squares in a particular way that the first group contain only one square each group contain 2 more squares than the previous one.

A visual proof for the above statement is given in the right side figure itself. The third group is colored in green. The 4th group contain a copy same third row (colored in green) and two more squares (colored in blue). This indicate that 4th group has two more squares than the 3rd one.

Similar to Algebraic Proof the above proof is for 5^2. It is obviously extendable to any square.