Sum of digits of a number

Some of you may be aware that sum of digits of a number always give the remainder produced when the number is divided by 9.

For example sum of digits of the number 29307634 = 2 + 9 + 3 + 0 + 7 + 6 + 3 + 4 = 34 and again the sum of digits 3 + 4 = 7. So when 2307634 is divided by 9 the remainder will be 7. Obviously there is no need to consider digits 0 when adding.

If we check with a calculator 29307634/9 = 3256403 + 7/9 which means remainder = 7. OK

When adding the digits for the easiness of addition we can avoid digits 9. In the above example itself if we avoid the second digit the addition become 2 + 3 + 7 + 6 + 3 + 4 = 25 and again the sum of digits 2 + 5 = 7. The result is same 7.

Similarly we can avoid any two or 3 digits that obviously sums to 9. In the above example digits 6 and 3 can be avoided. Hence 2 + 3 + 7 + 4 = 16 and final result is again 1 + 6 = 7.

Before moving to the proof, check this. When we add 9 to a single digit non-zero number the digits of the result is adjusted for maintaining the sum of digits. For example when 9 is added to 5 the result 14 cross 10 (become 2 digit number). When a 1 appear as the second digit the value of unit digit is reduced by 1. In other words when digit in the 10s place is incremented by 1 the digit in the unit place is reduced by 1. This in turn balances the sum of digits. So adding 9 do not alter the sum of digits.

Also one more point is to be taken care. For numbers that are multiples of 9 the remainder produced when it is divided by 9 should be 0. When adding the digits of such numbers the answer will be 9. This means the remainder is 9 which in turn mean the remainder is 0. So there is no logical error.

One logical thing that we may ask is that whether the rule of sum of digits is applicable for number systems other than Decimal? Naturally the answer should be YES. For example the sum of digits of a hexadecimal number is equivalent to remainder produced when this hexadecimal number is divided by 15. Why 15?, it is 1 less than the base of the number system. Take a random number 267830378723. The hexadecimal equivalent of this number is 3E5BEF40E3. Adding the digits = 3+E+5+B+E+F+4+0+E+3 = 53 and in turn = 5 + 3 = 8. That is, 8 is the remainder when 267830378723 is divided by 15.

Now coming to the proof:

Since the proof should be applicable for any integer we have to consider a general form of an integer. An integer with n+1 digits can be written as:

An An-1 An-2 . . . . A1 A0 ---- (1)

Where An is the left most digit, An-1 is the next digit and so on and A0 is the right most digit (unit place).

For example take a 5 digit number

15402

Here A0 = 2, A1 = 0, A2 = 4, A3 = 5 and A4 = 1.

In the general form of the integer represented in (1) above it is clear that all A0, A1 … An are single digit numbers less than or equal to 9. The real value of this number is:

$A0 + A1 \times 10 + A2 \times 100 + . . . + An \times 10^n$ --- (2)

So the sum of digits = A0 + A1 + A2 + . . . + An --- (3)

What we need to prove is that (2) and (3) will have same remainder when divided by 9. To prove this it is sufficient to prove that when (3) is subtracted from (2) we get a number that is fully divisible by 9. So

(2) – (3) = $A0 + A1 \times 10 + A2 \times 100 + . . . + An \times 10^n -$ A0 - A1 - A2 - . . . – An

$= A0 - A0 + A1\ (10 - 1) + A2\ (100 - 1) + \cdots . An\ (10^n - 1)$

$= A1\ (10 - 1) + A2\ (100 - 1) + \cdots . An\ (10^n - 1)$ --- (4)

The values shown in red colour are 9, 99, 999, 9999 . . . etc. It is very clear that these values are divisible by 9. So (4) is divisible by 9. This complete the proof.